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Forecasting Stock Prices Using Heston-Artificial Neural Network Model

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Abstract: Considering the evolution of financial globalization and the impacts of the global economic crisis, stock trading faces unprecedented fluctuations. The inherent volatility in stock prices has resulted in market uncertainty, prompting an interest among investors in reliable pricing models in order to maximize profits. To this end, researchers have continued to diligently refine stock pricing models to mitigate market uncertainty. One notable contender in this arena is the Heston model, conceived to remedy the limitations of the Black-Scholes model. The model embraces stochastic volatility, a departure from the constant volatility assumption underpinning the Black-Scholes model. However, the Heston model itself grapples with certain pivotal constraints, mainly the requisite precision in parameter calibration to produce a reliable estimate. Leveraging the current wave of technological advancement, this study uses an Artificial Neural Network (ANN) as a substitute for simulating different volatility parameters in the Heston model. This approach culminates in the construction of a hybrid semi-parametric forecasting model termed the Heston-ANN model. The study applies this model to datasets of three distinct stocks: BA, IBM, and GOLD. Through graphical analysis and the evaluation of different model performance metrics including Mean Absolute Percentage Error, Mean Absolute Error, and Mean Squared Error, the study compares the hybrid model to the original Heston model. The results reveal that the Heston-ANN model yields more accurate forecasts when juxtaposed with its precursor, the original Heston model. The synergy between the Heston model and ANN makes the hybrid model a more robust solution for forecasting stock prices.

Keywords: Stock Pricing, Artificial Neural Network (ANN), Stochastic Volatility, Stochastic Differential Equation (SDE)

1. Introduction

Investing in a particular economic sector can be daunting and, at the same time, very rewarding. One way an investor can invest is by buying and selling stocks of a listed company in a securities exchange. A stock is a security that highlights partial ownership in a company. The common goal among all investors is to make profits. However, buying and selling stocks gives rise to volatile stock prices. The volatility makes it challenging to accurately predict the stock price movement, and forecasting and identifying the right time to buy or sell stocks is challenging [1]. Many studies have been conducted extensively to develop a successful approach for forecasting stock prices [2]. Stock price movement is influenced by economic, social, political, and psychological factors which interact in a multiplex way. Most investors and financial advisors are interested in knowing the future direction of the movement of stock prices. This knowledge gives them the necessary guidance in developing profitable market trading strategies.

Market volatility measures the extent to which the market's overall value varies within a given period. The higher the risk, the higher the returns. Thus, no investor would want a market where prices do not fluctuate. Forecasting the price of a stock means predicting or estimating a future price. However, future stock prices are stochastic. Despite that uncertainty, Chandan Sengupta holds that the prices adhere to specific criteria generated from past data and the knowledge of share prices [3]. The main goal of predicting stock prices is to attain the best results while reducing erroneous price forecasts [4]. Therefore, investors need a reliable method of forecasting volatility and, thus, stock prices to make informed investment decisions.

This article aims to combine one of the outstanding option pricing models, the Heston model, with ANN to develop a hybrid forecasting model termed the Heston-ANN model. The synergy between the stochastic volatility framework of the Heston model and ANN's computational power makes the hybrid Heston-ANN model a robust solution for forecasting stock prices. The second section provides the Literature review, the next section outlines the methodology used to achieve the study objectives; the fourth section highlights and discusses the results of the study; and the final section offers the conclusion and recommendations.

2. Literature Review

The late 20^{th} century saw the development of mathematical models for predicting stock price movements, such as the Geometric Brownian Motion (GBM) and the Black-Scholes model. GBM adopts an algorithm that begins by calculating the return value to calculate the confidence level [5]. The Black-Scholes model is also critical in stock pricing. It has successfully estimated the value of derivatives while utilizing other investment instruments, considering the influence of time and other critical risk factors. Such pricing models are essential for investors in the financial marketplace while determining and predicting the prices of various options and when trading with stocks. As G Preethi and B Santhi highlighted, securities present volatile prices throughout the trading day [4]. Therefore, the stock pricing models help analyze and integrate the variables causing fluctuations to identify the most appropriate stock to trade in.

However, the stock pricing models developed so far present various assumptions and limitations, which still need to be revised to predict stock prices accurately. The Black-Scholes, for instance, presents errors and incorrect pricing of securities due to its reliance on constant volatility [6]. In the real market, volatility is not constant; it is stochastic. Consequently, such critical limitations have called for a constant review of existing models and the development of improved models, as this research study aims. Such approaches ensure stock prices are more predictable, allowing profitable interactions with the stock market.

As created in 1993 by Steven Heston, the Heston model is a stochastic volatility model used over time primarily for pricing European Options. It relies on stochastic volatility, contrasting the Black-Scholes model, which is based on the premise of constant volatility. The Black-Scholes model's success arises from how it relates to the distribution of spot returns for the cross-sectional features of option prices [7]. Heston generalizes the Black-Scholes model while maintaining that property.

The fact that the volatility is arbitrary, as opposed to constant, is the most outstanding feature that makes the Heston model more reliable for predicting stock prices than the other models. The stochastic volatility is implemented by generating random numbers in a Monte Carlo engine. This iteration is performed by the Monte Carlo simulation as numerous times as necessary, producing a probability distribution of all probable results [8]. Another key feature of the model is that it permits the volatility to be correlated to the stock price, which is critical when forecasting stock prices. This model also brings out a closed-form solution. As Pierre Gauthier and Dylan Possama¨i highlighted, the answer from this model is developed from a recognized mathematical computation [9]. Notably, it also brings out vitality as reverting to the mean and does not consider stock prices because it adopts a lognormal probability distribution as a prerequisite. As confirmed by Fernando Ormonde Teixeira, the Heston model is also related to the volatility smile model, which illustrates a graphical representation with a range of identical expiration dates [10]. The model is key because it presents increasing volatility as an option moves from in-the-money to out-of-the-money. These aspects make the Heston model more reliable for predicting stock prices than other models.

Nevertheless, the Heston model has some limitations. The model involves parameters that require careful calibrations to develop a reliable estimate. Additionally, the Heston model suffers a critical limitation while forecasting option prices for short-term options. The failure to predict short-term option prices is due to the model's failure to capture the high implied volatility (the expected market volatility from the time of purchase of the option to its expiry) [11]. Also, a noticeable limitation is that the Heston model is more complex compared to the Black-Scholes model, which hinders investors from using it. Researchers continue to address these shortcomings to improve the model's efficiency.

Charlotte et al. focused on improving the Heston model by incorporating it with long-term jump diffusion to address the challenge of predicting short-term maturities [11]. The study compared the findings from the developed version (Heston model with a jump) to the original Heston model and the Black Scholes model based on short maturities. Considering the MSE as the performance metric, the Heston model with a jump outperformed both the original Heston model and the Black-Scholes model for short-term maturities. The improvement was notable, showing a 58.08\% reduction in error compared to the original Heston model and a 47.3\% reduction compared to the Black-Scholes model.

Although the Heston model was primarily developed for the financial market, recent research has shown that the model application can extend to other industries. Bianca Reichert conducted a comprehensive review of existing literature to determine the applicability of the model in the energy sector for predicting energy generation [12]. The review encompassed a textual corpus of 25 papers. The findings indicated that the Heston model is indeed suitable for energy generation prediction due to its ability to provide a closed-form solution and effectively model stochastic volatility, which facilitates forecasting the average energy generation values.

While the stock market system remains chaotic in terms of drastic and unprecedented changes, recent technological advancements have led to a revolutionized forecasting era. Such advancements are critical in helping investors make immediate and well-informed decisions. As Luca Di Persio and Oleksandr Honchar suggest, an investor desires to forecast the stock price accurately and make a sale before the value declines or purchase the stock before the prices increase [13]. The introduction of machine learning in stock pricing has been a significant and valuable technological advancement in the stock market. It allows for predicting future stock prices and value with enhanced accuracy, which has been a significant hindrance to making desirable profits from the stock market [14]. Machine learning has enabled the development of Artificial Neural Networks (ANN), which incorporate technical analysis, a key component of forecasting in the stock market.

ANN is designed to comprehend complicated structures which traditional machine-learning algorithms cannot undertake. Comparable to the human brain, ANN exists in a complex and complicated form of interconnectivity. It is built through algebraic equations that direct information to a particular model. The use of ANN allows for reliable modeling techniques of datasets. It is a mesh of numerical equations with which variables are applied and processed to obtain one or multiple outputs. The quality of a model hinges on its accuracy in making predictions [14]. The back-propagation algorithm, which serves as the foundation for ANN, enables the network to excel in achieving this goal, thus establishing it as a highly valuable learning model. ANN's capability to effectively model data with high volatility and non-constant variance further enhances its ability to produce more accurate and reliable predictions. Therefore, using ANN has demonstrated efficiency in forecasting financial time series, particularly when dealing with data that exhibits significant volatility.

Artificial neural networks are considered more viable compared to traditional statistical methods since they can approximate any non-linear function with considerable accuracy without making any prior assumptions about their nature. More so, neural networks can bear the market's noisy and chaotic components while remaining less sensitive to error terms, assumptions, and limitations [15]. This study uses the popular feed-forward multilayer network, consisting of three layers: input, hidden, and output. The model then uses the Rectified Linear Unit (ReLU) activation function to map non-linearly from the past observations (input values) to the future value (output). ReLU is chosen over other activation functions because it is very fast in computation and does not have a saturation point, which helps gradient descent.

3. Methodology

3.1. The Heston Model

The Heston model arises from the fact that an asset's volatility is stochastic. The model is given by the stochastic differential equations (SDEs) below. The SDEs represent the stock price and the volatility diffusion processes given a probability measure $\mathbb{P}$.

We assume the stock price follows the following diffusion at time t

$$dS_t = \mu S_t dt + \sqrt{v_t} S_t dw_1(t) \tag{1}$$

where $dw_1(t)$ is a Wiener process

μ is the drift parameter

$\sqrt{v_t}$ is the volatility

The volatility follows an Ornstein-Uhlenbeck process [16]

$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}dw_2(t) \tag{2}$$

where

$dw_2(t)$ is a Wiener process

$\sqrt{v_t}$ is the volatility

θ is the long-run mean of the variance

κ is the speed of mean reversion

σ is the volatility of volatility

$$dw_1(t)dw_2(t) = \rho dt, \rho\epsilon[-1,1]$$

The model assumes a constant correlation ρ between the two Brownian motions. The correlation between the asset price and the volatility gives a closed-form solution and enables the incorporation of stochastic interest rates into the model.

For simplicity, let r be the risk-neutral drift. Then equation (1) can be written as:

$$dS_t = rS_t dt + \sqrt{v_t} S_t dw_1(t) \tag{3}$$

One would need to compute the numerical value of $\mathbb{E}[f(S_t)]$ for a payoff function $f: \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ where a stochastic function refers to the underlying stock S:

$$dS_t = a(S_t, t)dt + b(S_t, t)dW_t \tag{4}$$

Monte Carlo simulation can be used through the discretization of the time interval and simulation of the state process dynamics on the discrete-time grid. In such a case, it would be important to use a discrete-time approximation of equation (4) to derive a Monte Carlo estimate of $\mathbb{E}[f(S_t)]$. The simulation gives numerous sample trajectories of the state variables such as interest rates, volatility, and stock price [17].

This study uses the log stock prices of the underlying asset $X_t = \ln[S_t]$ to smoothen the discretization process.

Applying Ito's lemma to equation (3) gives:

$$dX_t = r - \frac{1}{2}v_t dt + \sqrt{v_t} dW_1(t) \tag{5}$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t} dW_2(t) \tag{6}$$

The study uses the natural log of variance to avoid negative values in the continuous-discrete time transition.

Using Ito's lemma again, we get the following:

$$d\ln(v_t) = \frac{1}{v_t}\left(\kappa(\theta - v_t) - \frac{1}{2}\sigma^2\right)dt + \sigma\frac{1}{\sqrt{v_t}}dW_2(t) \tag{7}$$

Executing the Euler discretization technique results in the discrete-time solutions below:

$$\ln(S_{t+\Delta t}) = \ln S_t + \left(r - \frac{1}{2}v_t\right)\Delta t + \sqrt{v_t}\sqrt{\Delta t}\epsilon_{S,t+1} \quad (8)$$

$$\ln(v_{t+\Delta t}) = \ln v_t + \frac{1}{v_t}\left(\kappa(\theta - v_t) - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\frac{1}{\sqrt{v_t}}\sqrt{\Delta t}\epsilon_{v,t+1} \quad (9)$$

Taking exponentials gives the Heston model:

$$S_{t+\Delta t} = S_t \cdot \exp\left(r - \frac{1}{2}v_t\right)\Delta t + \sqrt{v_t}\sqrt{\Delta t}\epsilon_{S,t+1}) \quad (10)$$

$$v_{t+\Delta t} = v_t \cdot \exp(\frac{1}{v_t}\left(\kappa(\theta - v_t) - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\frac{1}{\sqrt{v_t}}\sqrt{\Delta t}\epsilon_{v,t+1}) \quad (11)$$

3.2. Artificial Neural Network

This study used a one-hidden layer feed-forward neural network optimized using the adaptive moment estimation (Adam optimization) algorithm. The algorithm is founded on gradient descent. It adapts the learning rate for all the neural network's weights using estimations of the gradient's first and second-order statistical moments. Adam optimizer is chosen over other optimizers because it needs fewer parameters for tuning and is faster in computation.

The neural network takes an input vector of dimension p, $X = (x_{t-1}, x_{t-2}, \ldots, x_{t-p}) \in \mathbb{R}^p$, where p denotes the number of predictors, and then builds a non-linear function, $f(x)$, to predict the response Y. Denote, H as the hidden layer, $\psi(x)$ as the transfer or activation function, ω_{hi} as the weights that connect the input and the hidden layer where $h \in \{1, \ldots, H\}$ and $i \in \{1, \ldots, p\}$, α_{oh} as the weights connecting the hidden and output layer, and β_h and β_o as the bias or offset input to the hidden and output layer respectively. The bias always has a value of 1 at the beginning of the training and has its own weight to ensure there is an activation in the neuron, even if the inputs are all zeros.

The p features $x_1, x_2, \ldots, x_p$ make up the units in the hidden layer. At the hth hidden node, the neural network has the form,

$$f_h(X) = \beta_h + \sum_{i=1}^{p} \omega_{hi}\, x_i \quad (12)$$

At the h^{th} hidden node, the activation function, $\psi(z)$, squashes the input values into a smaller range depending on the activation function chosen. This study uses the rectified linear unit (ReLU) activation function, which takes the form:

$$\psi(z) = \begin{cases} 0 & x \leq 0 \\ x & x > 0 \end{cases} \quad (13)$$

The activation function returns zero for negative values of the input and returns the value back for positive input values. Its graphical representation is as in the figure below:

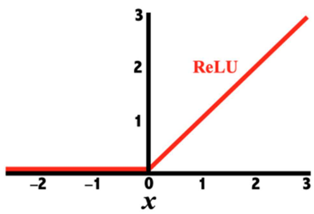


Image source: https://mriquestions.com/convolutional-network.html

Figure 1. *ReLU Activation Function.*

At the output node, the neural network takes the form:

$$f_o(X) = \beta_o + \sum_{h=1}^{H} \alpha_{oh}\, \psi(f_h(X)) \quad (14)$$

The final output is given by:

$$\begin{aligned} f(X) &= \psi(f_o(X)) \\ &= \psi\left(\beta_o + \sum_{h=1}^{H} \alpha_{oh}\, \psi(f_h(X))\right) \\ &= \psi[\beta_o + \sum_{h=1}^{H} \alpha_{oh}\, \psi(\beta_h + \sum_{i=1}^{p} \omega_{hi}\, x_i)] \end{aligned} \quad (15)$$

This study uses equation (16), as proposed by C.G. Looney, to determine the number of nodes, H, in the hidden layer [18].

$$H = ||1.7 * \log 2(n)|| + 1 \quad (16)$$

Where n is the size of each input vector (predictor) of the training dataset.

The study uses ANN to predict v_{t+1}. The input volatility is calculated using the PK volatility estimator given by:

$$\hat{\sigma}^2{}_{PK} = (4\ln 2)^{-1} \cdot [\ln\left(\frac{H_t}{L_t}\right)]^2 \quad (17)$$

where H_t and L_t denote the high and low stock prices at day t, respectively.

This study also applies the sum of squared error (SSE) method to train the neural network where the weights between the target, Y, and the final output, $f(X)$, are minimized. Let θ represent the set of all the weight parameters, that is, α_{oh} and ω_{hi} of the network. The SSE is defined as:

$$\begin{aligned} S^2(Y, X; \theta) &= \sum_{i=1}^{p}\left(y_i - f(x, \theta)\right)^2 \\ &= \sum_{i=1}^{p}[\, y_i - \psi[\beta_o + \sum_{h=1}^{H} \alpha_{oh}\, \psi(\beta_h + \sum_{i=1}^{p} \omega_{hi}\, x_i)]]^2 \end{aligned} \quad (18)$$

3.3. The Heston-ANN Model

Let S_t and S_{t+1} denote the stock prices at time t and $t + 1$ respectively.

$$v_t = \beta_o + \sum_{h=1}^{H} \alpha_{oh}\, \psi(f_h(X)) \quad (19)$$

$$dS_t = \mu S_t dt + f_0(x) S_t dW_t \quad (20)$$

$$\frac{dS_t}{S_t} = \mu dt + f_0(x) dW_t \quad (21)$$

Consider a function $G(t, S_t) = \ln S_t$, where G is twice differentiable on S_t and once on t. For a Heston diffusion process, the Ito's lemma is given by [19]:

$$dG = \left[a\frac{\partial G}{\partial S_t} + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial S_t^2}b^2\right]dt + b\frac{\partial G}{\partial S_t}dB_t \tag{22}$$

where

$$a = \mu S_t,\ b = \sigma S_t,\ \frac{\partial G}{\partial S_t} = \frac{1}{S_t},\ \frac{\partial^2 G}{\partial S_t^2} = -\frac{1}{S_t^2},\ \text{and } \frac{\partial G}{\partial t} = 0$$

By inserting the above equalities into equation (22), we get

$$dG = \left[\mu - \frac{1}{2}f_0(x)^2\right]dt + f_0(x)dB_t \tag{23}$$

Since $G(t, S_t) = \ln S_t$, equation (23) can be written as:

$$d(\ln S_t) = \left[\mu - \frac{1}{2}f_0(x)^2\right]dt + f_0(x)dB_t \tag{24}$$

$$\int_0^t d(\ln S_t) = \int_0^t \left[\mu - \frac{1}{2}f_0(x)^2\right]dt + \int_0^t f_0(x)dB_t \tag{25}$$

Assuming we are interested in the solution at t and recalling that $B_0 = 0$ since the Wiener process starts at zero by definition, we integrate equation (25) from 0 to t to obtain,

$$\ln S_t - \ln S_0 = \left[\mu - \frac{1}{2}f_0(x)^2\right]t + f_0(x)[B_t - B_0] \tag{26}$$

$$\Rightarrow \ln S_t = \ln S_0 + \left[\mu - \frac{1}{2}f_0(x)^2\right]t + f_0(x)B_t \tag{27}$$

where $B_t \sim N(0, t)$.

We are interested in the annual returns, implying that $t = 1$ represents 1 year, and $\Delta t = \frac{1}{252}$ represents 1 day, assuming 252 trading days in a year. Let $\widehat{m_d}$ represent the mean of daily log returns, and let $\widehat{m_a}$ and $\widehat{\sigma_a}^2$ denote the annual mean and variance of log returns respectively. In addition, B_t can be represented as the square root of t times a standard normal, that is, $B_t = (\sqrt{t})Z$, where $Z \sim iidN(0,1)$. Also, t can be written in terms of the size of the time step, that is, Δt. Exponentiating both sides of equation (27) and rearranging, we obtain the Heston-ANN model given as:

$$S_{t+\Delta t} = S_0 \exp\left[\left(\hat{\mu} - \frac{1}{2}f_0(x)^2\right)\Delta t + f_0(x)(\sqrt{\Delta t})Z\right] \tag{28}$$

where

$$\hat{\mu} = \widehat{m_a} + \frac{\widehat{\sigma_a}^2}{2}$$

$$\widehat{\sigma_a}^2 = \left[\frac{1}{n-1}\sum_{i=1}^{t}(r_i - \widehat{m_d})^2\right] * 252$$

$$r_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$$

$$\widehat{m_d} = \frac{1}{n}\sum_{i=1}^{n} r_i$$

$$\widehat{m_a} = \widehat{m_d} * 252$$

3.4. Evaluation of Model Performance

This study compared the performance of Heston-ANN and the original Heston model both graphically and by computing the Mean Absolute Percentage Error (MAPE), Mean Squared Error (MSE), and Mean Absolute Error (MAE) given by the equations below:

$$MAPE = \left(\frac{1}{n}\sum_{i=1}^{n}\left|\frac{S_i - \hat{S}_i}{S_i}\right|\right) * 100 \tag{29}$$

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(S_i - \hat{S}_i)^2 \tag{30}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|S_i - \hat{S}_i\right| \tag{31}$$

4. Results and Discussions

4.1. Data Sources and Description

This study used secondary data from Yahoo Finance [1]. In particular, the study considered three stocks under different sectors in the Global Industry Classification.

Standard (GICS), namely: The International Business Machines Corporation (IBM),

The Boeing Company (BA), and Barrick Gold Corporation (GOLD) under technology, industrials, and materials sectors respectively. The period considered was from January 03, 2011, to January 03, 2022. The study focuses on forecasting future stock prices using the Heston-ANN model. Data analysis was done using Python. The dataset was split into train and test data at 70:30, and the train data was further split into train and validation data at 80:20.

4.2. Forecasting Stock Prices Using the Heston and Heston-ANN Models

4.2.1. Graphical Comparison of the Models Against Actual Prices

The stock price forecasts obtained through both the Heston and the Heston-ANN models were plotted against the actual prices observed in the market for six different time steps: 7, 14, 21, 28, 35, and 42 days as shown in the plots below.

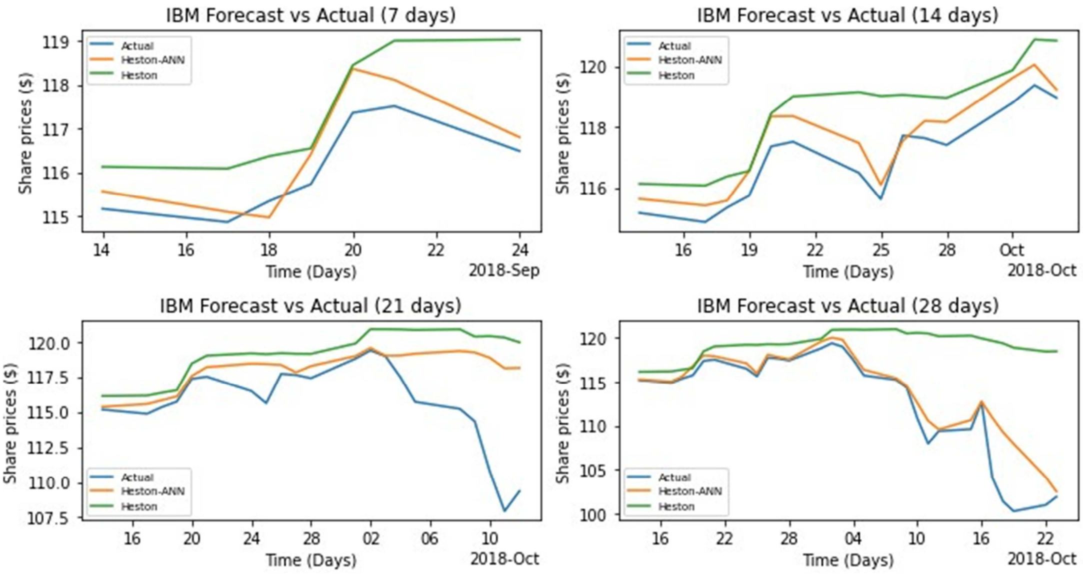


Figure 2. *IBM Forecasts vs. Actual prices (7-28 days).*

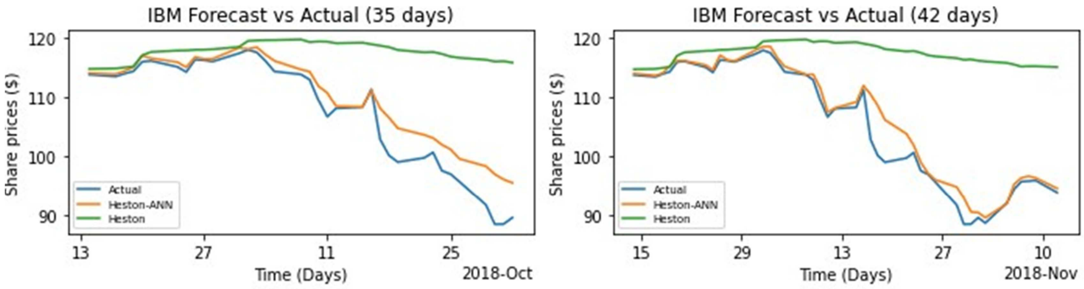


Figure 3. *IBM Forecasts vs. Actual prices (35-42 days).*

Figures 2 and 3 show that the Heston-ANN model performed better than the Heston model for the six time steps for IBM stock prices. The plots also show that the forecasts of the Heston model deviated further from the actual prices with the increase in the days considered. However, while the Heston-ANN model outperformed the Heston model, both models needed to be more accurate in forecasting IBM stock prices. The low accuracy can be attributed to the high price fluctuation of IBM share prices.

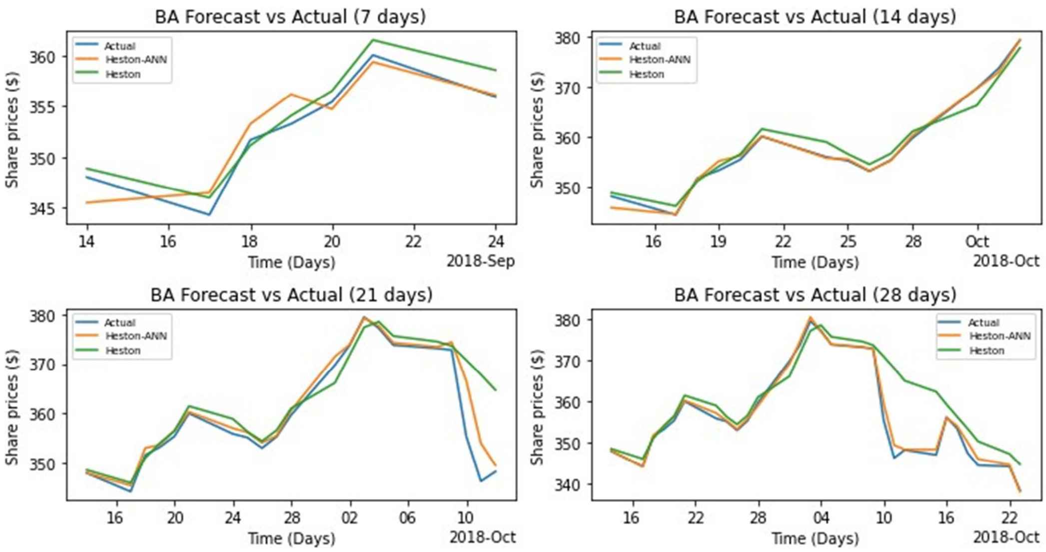


Figure 4. *BA Forecasts vs. Actual prices (7-28 days).*

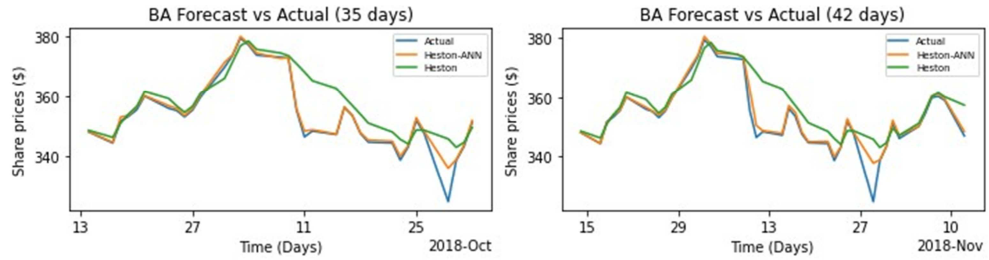


Figure 5. *BA Forecasts vs. Actual prices (35-42 days).*

According to Figures 4 and 5, the forecasts of the two models were close to the actual BA stock prices. However, the Heston-ANN did better than the Heston model for all the six time steps.

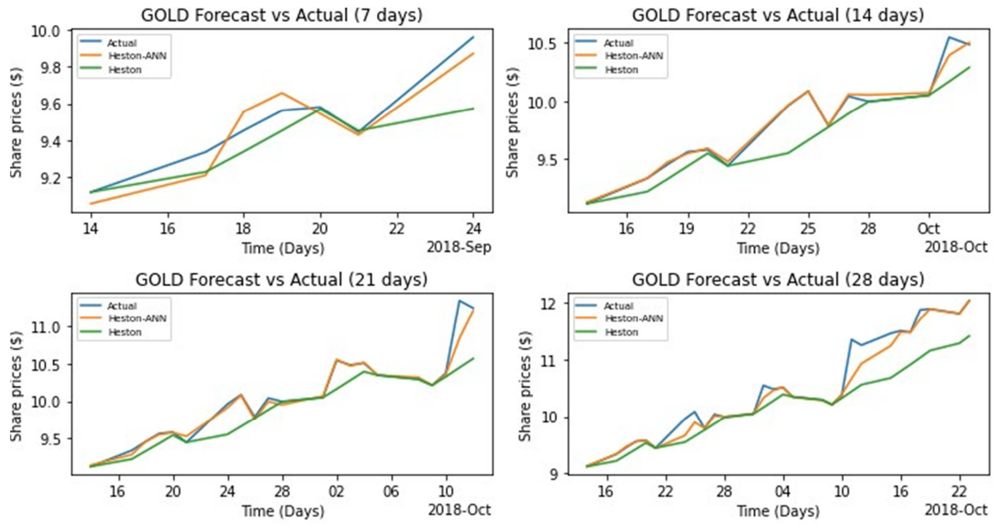


Figure 6. *GOLD Forecasts vs. Actual prices (7-28 days).*

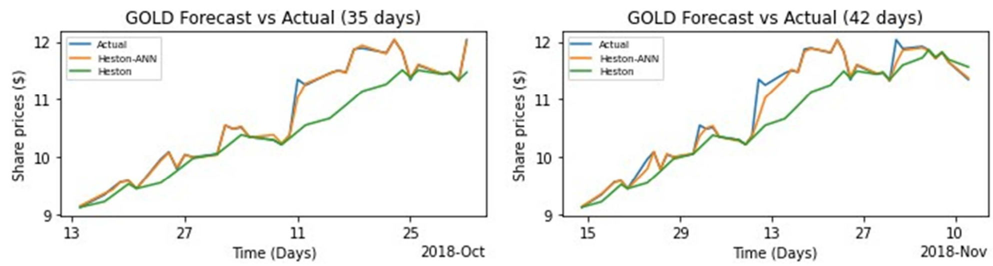


Figure 7. *GOLD Forecasts vs. Actual prices (35-42 days).*

Figures 6 and 7 showed that the Heston-ANN forecasts were more accurate than the Heston model for GOLD stock prices for all time steps considered. The Heston curve also deviated further from the actual prices over time.

4.2.2. Mean Absolute Percentage Error (MAPE)

Table 1 gives the MAPE values for both Heston-ANN and Heston models for the various time steps considered. The MAPE values were lower for the hybrid model than the Heston model, except for BA stock prices at seven days, showing that Heston-ANN performed better than the Heston model. The MAPE values increased with the increase.

Table 1. *Comparison of MAPE values in the number of days for the Heston model showing that the model performance deteriorated over time.*

Stock	Model	7 days	14 days	21 days	28 days	35 days	42 days
IBM	Heston	0.01143	0.01342	0.03193	0.06088	0.10115	0.12406
	H-ANN	0.00396	0.00614	0.00756	0.01738	0.02451	0.01394
BA	Heston	0.00367	0.00427	0.01084	0.01261	0.01343	0.01256
	H-ANN	0.00439	0.0015	0.01076	0.00226	0.00261	0.00335
GOLD	Heston	0.01083	0.01384	0.01696	0.02759	0.02498	0.02273
	H-ANN	0.00788	0.00348	0.00531	0.00386	0.02325	0.00469

4.2.3. Mean Squared Error (MSE)

Table 2. *Comparison of MSE values.*

Stock	Model	7 days	14 days	21 days	28 days	35 days	42 days
IBM	Heston	2.01859	2.86932	24.00072	77.76394	179.1156	232.7177
	H-ANN	0.2978	0.69987	1.41707	9.90279	11.80713	5.97307
BA	Heston	2.10803	2.95889	48.40193	50.53428	55.88167	50.42571
	H-ANN	3.31257	0.74296	48.20019	1.57002	4.1255	6.17164
GOLD	Heston	0.02664	0.04185	0.09042	0.18884	0.16808	0.14779
	H-ANN	0.00692	0.00449	0.0211	0.01275	0.11694	0.01778

According to Table 2, the Heston-ANN model had lower MSE values compared to the Heston model implying better performance by the hybrid model.

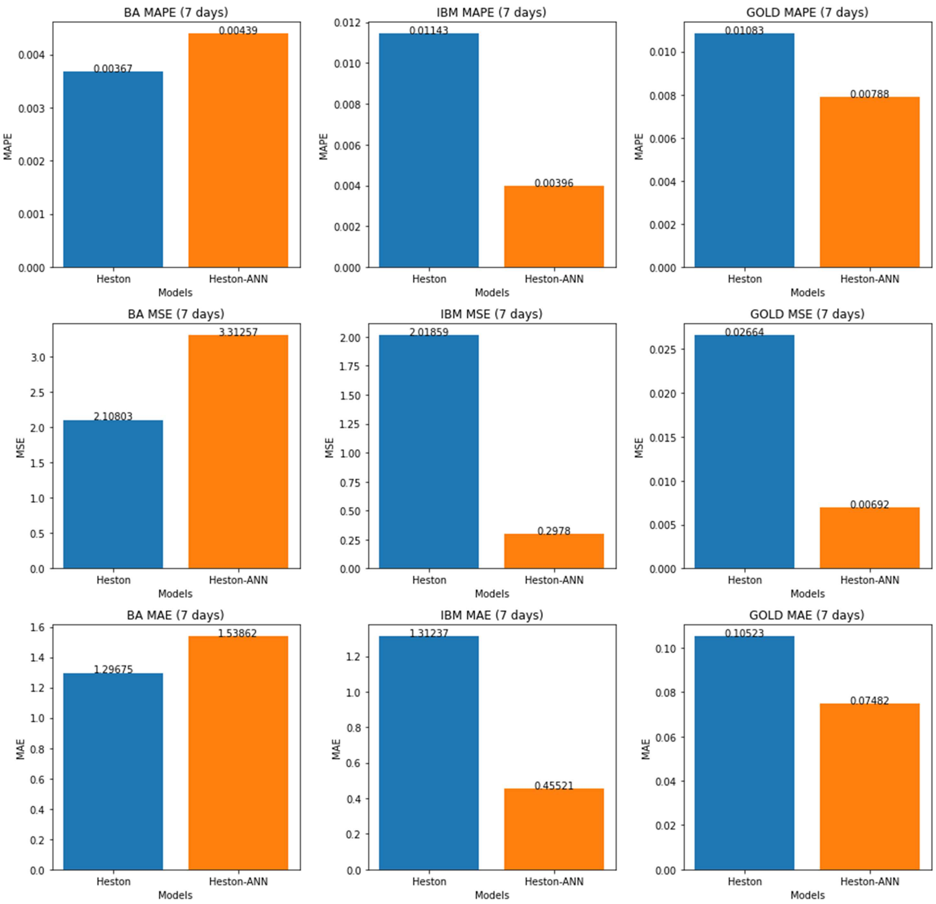
4.2.4. Mean Absolute Error (MAE)

Table 3. *Comparison of MAE values.*

Stock	Model	7 days	14 days	21 days	28 days	35 days	42 days
IBM	Heston	1.31237	1.5505	3.56689	6.44034	9.9913	12.04768
	H-ANN	0.45521	0.7086	0.84667	1.80492	2.40848	1.39605
BA	Heston	1.29675	1.53539	3.83795	4.43073	4.66329	4.36343
	H-ANN	1.53862	0.53147	3.83531	0.7985	0.8957	1.16166
GOLD	Heston	0.10523	0.13899	0.17899	0.30799	0.28064	0.25607
	H-ANN	0.07482	0.03517	0.05653	0.04179	0.31935	0.05239

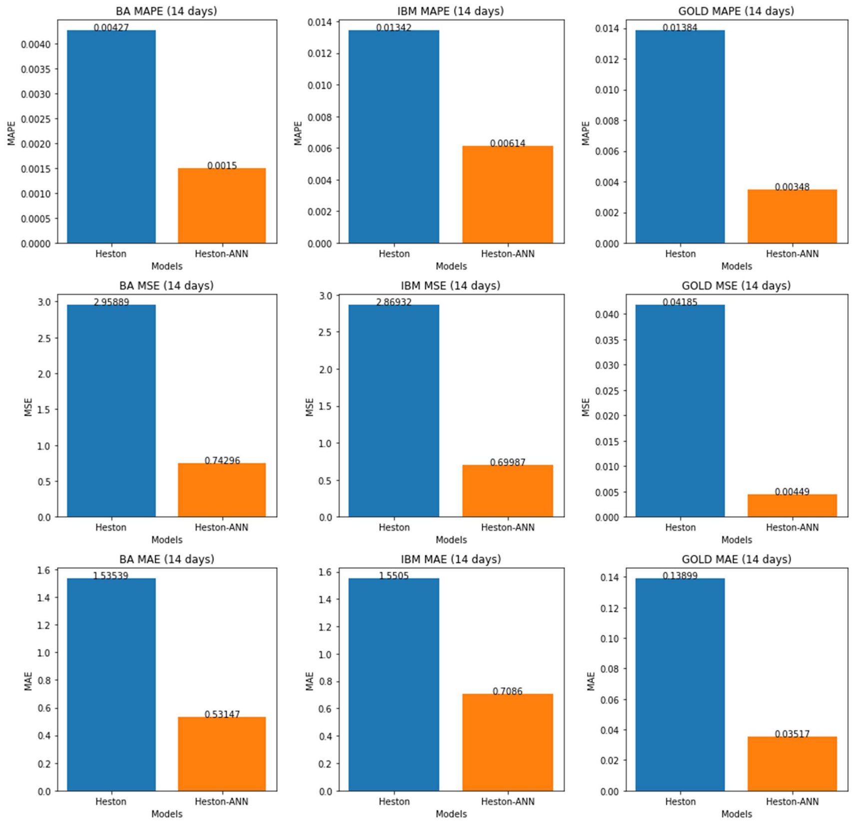
As seen in Table 3, the MAE values were consistent with the MAPE and MSE values.

The values are higher for the Heston model than the Heston-ANN model. The values also show a general increase for the Heston model, implying that the model performance worsens over time.



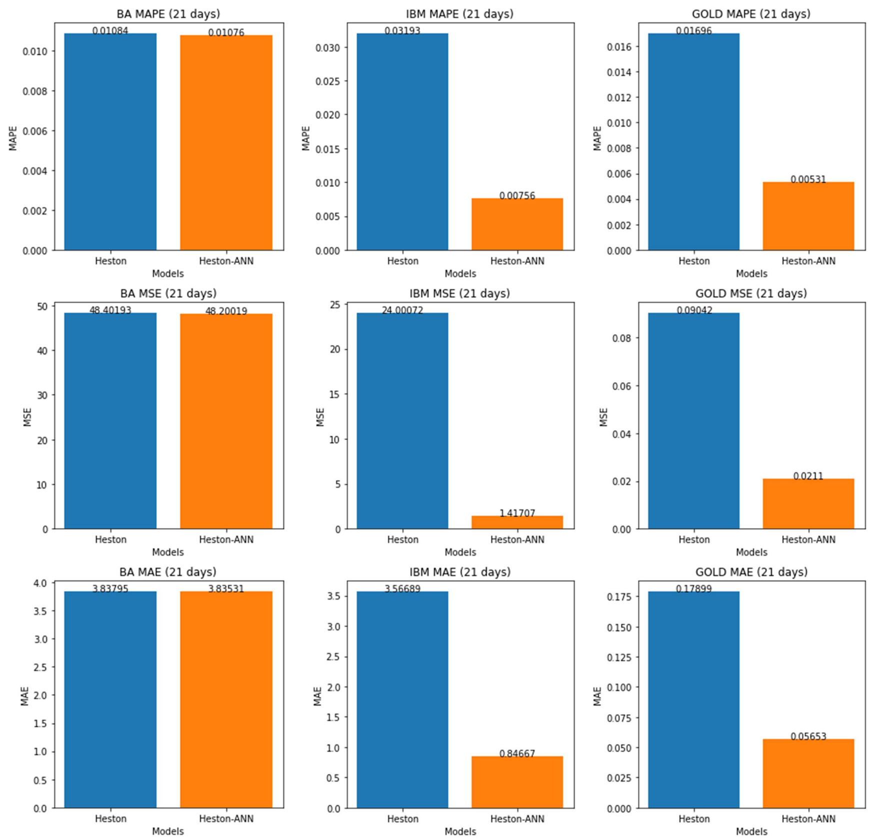
(a) BA (b) IBM (c) GOLD

Figure 8. *7 days MAPE, MSE, and MAE.*



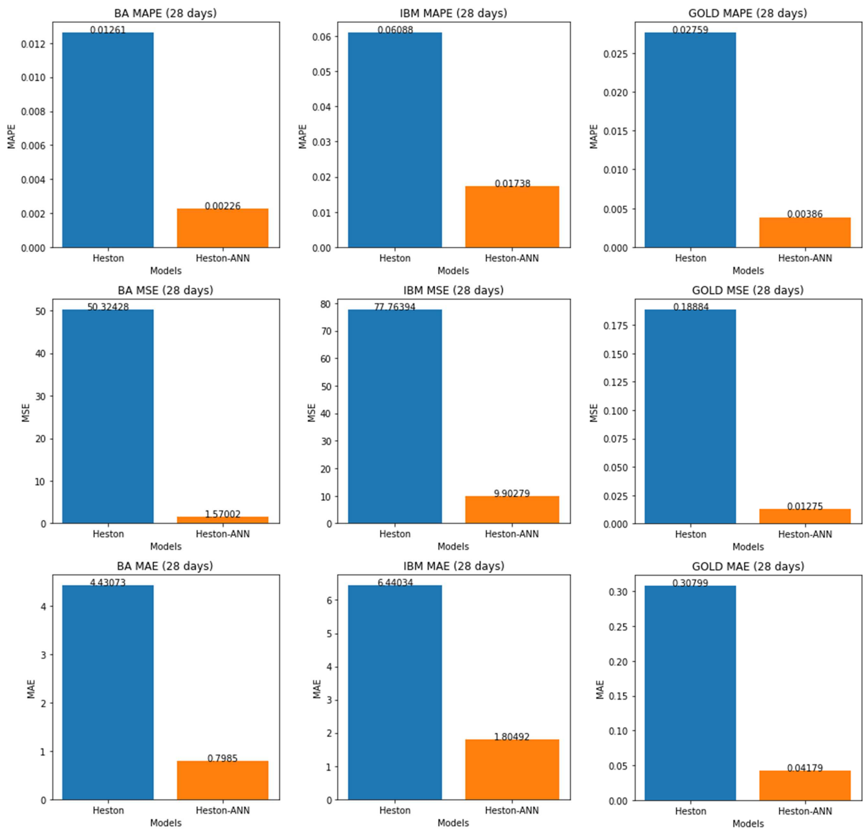
(a) BA (b) IBM (c) GOLD

Figure 9. *14 days MAPE, MSE, and MAE.*



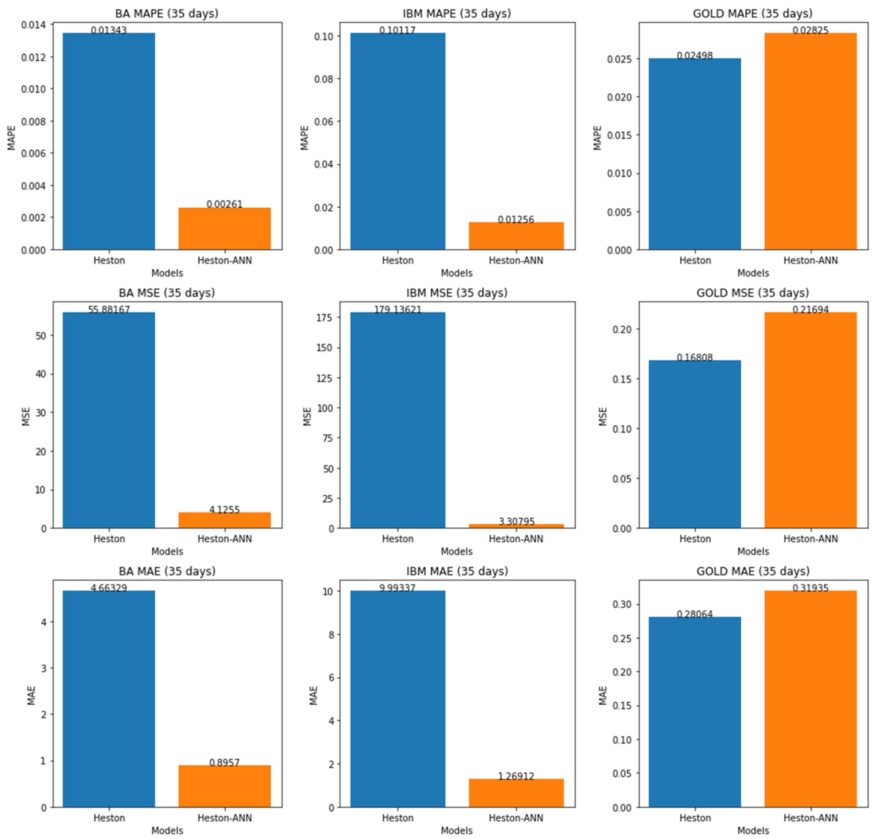
(a) BA (b) IBM (c) GOLD

Figure 10. *21 days MAPE, MSE, and MAE.*



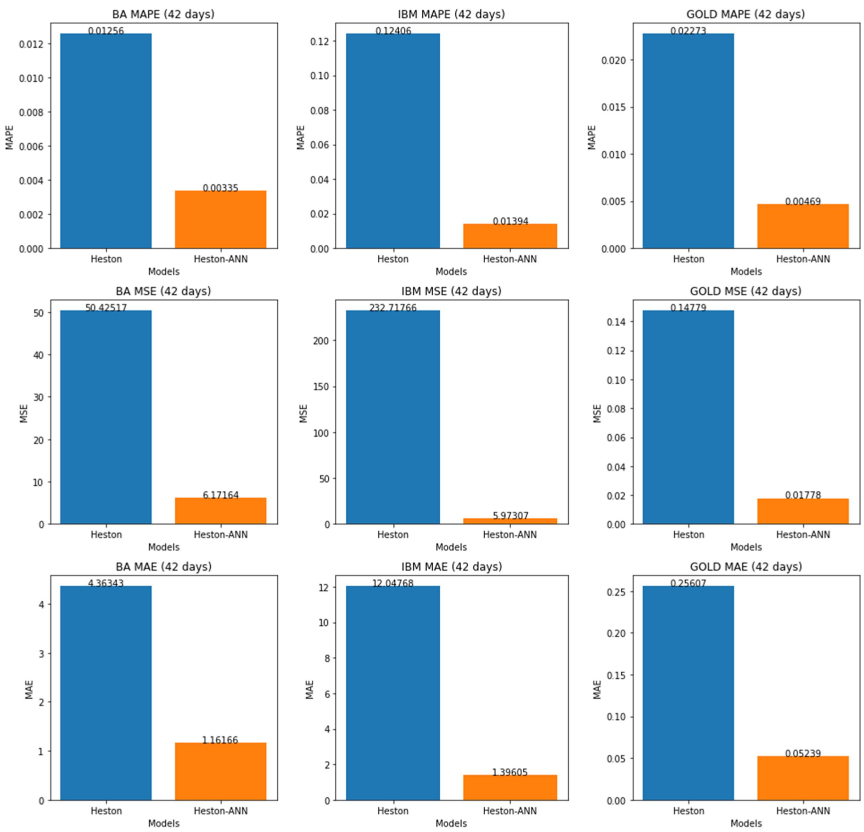
(a) BA (b) IBM (c) GOLD

Figure 11. *28 days MAPE, MSE, and MAE.*



(a) BA (b) IBM (c) GOLD

Figure 12. *35 days MAPE, MSE, and MAE.*



(a) BA (b) IBM (c) GOLD

Figure 13. *42 days MAPE, MSE, and MAE.*

Figures 8, 9, 10, 11, 12, and 13 graphically represent the three performance measures: MAPE, MSE, and MAE for the six time steps. According to the plots, the MAPE, MSE, and MAE values were lower for the hybrid model than the Heston model showing that Heston-ANN performed better than the Heston model. However, the Heston model outperformed the Heston-ANN model for BA stock at seven days which is seen as an outlier. The MAPE values increased with the increase in the number of days for the Heston model showing that the model performance declined over time. The results showed that the Heston-ANN model performed better than the Heston model for all three stocks.

5. Conclusions and Recommendations

In response to the challenges posed by existing stock pricing models and to investors' need for a reliable pricing model, this study embraced technological advancements by introducing an innovative paradigm - the integration of an Artificial Neural Network to estimate volatility within the Heston model. This approach resulted in the creation of the Heston-ANN semi-parametric hybrid model. The empirical application of this model is executed on datasets pertaining to three distinct stocks: BA, IBM, and GOLD. Through graphical analysis and a diverse spectrum of model performance metrics, this study conducted a comprehensive comparative evaluation between the hybrid Heston-ANN and the original Heston models. The compelling outcomes revealed the Heston-ANN model's superiority over the original Heston model. The amalgamation of the Heston model's stochastic volatility framework with the computational power of ANN makes the Heston-ANN model a more reliable and effective forecasting tool in stock trading.

In light of the evolving challenges faced by investors in stock trading, the findings of this study offer valuable insights and open avenues for further research. The successful integration of ANN into the Heston model has demonstrated the potential for enhanced forecasting accuracy in forecasting stock prices. Building upon this synergy, future studies could delve deeper into exploring different machine learning techniques, such as deep learning architectures, to further augment the predictive capabilities of financial models. Additionally, extending the scope of this research to encompass a wider array of industries, markets, and economic conditions could provide a comprehensive evaluation of the proposed model's adaptability. Moreover, as financial markets continue to evolve in response to global economic shifts and technological advancements, ongoing investigations into refining model calibration methodologies are recommended to address the inherent complexity of the Heston model's parameters. By continually refining and expanding upon the foundations established in this study, researchers and practitioners alike can contribute to the development of more accurate and reliable tools for navigating the intricacies of stock market forecasting and investment decision-making.

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1 Secondary data obtained from https://finance.yahoo.com/